WCCM V
Fifth World Congress on
Computational Mechanics
July 7-12, 2002, Vienna, Austria
Eds.: H.A. Mang, F.G. Rammerstorfer,
J. Eberhardsteiner

Control parameters for the analysis and visualization of FE results into a collaborative engineering environment

L. Fine, J-C. Léon*

Laboratory of Soils, Solids, Structures – Integrated Design Project
Domaine Universitaire, BP 53, 38041 GRENOBLE Cedex 9, France
e-mail: [Lionel.Fine,Jean-Claude.Leon]@hmg.inpg.fr

A. Gérussi, G-P. Bonneau

Laboratory of Modelisation and Computation – IMAG
Domaine Universitaire, BP 53, 38041 GRENOBLE Cedex 9, France
e-mail: [Alexandre.Gerussi,Georges-Pierre.Bonneau]@imag.fr

Key words: FE results, visualization, collaborative engineering, wavelets, polyhedron decimation

Abstract

Currenlty, FE meshes form the basic representation for the analysis and visualization of 3D FE analysis results. In this case, the triangulations used for visualization purposes are colored in accordance with the values issued from the resolution phase. As FE models get larger (hundreds of thousands and over), their 3D visualization become more and more difficult on high performance graphic workstations and cannot be performed on low cost computers. In the context of collaborative engineering, such models can hardly be exploited during digital design reviews (with or without immersion into a VR environment) and cannot be attached to emails and digital reports. In addition, large FE models reduce the understanding of phenomena since 3D animations and interactive analysis on high performance workstations cannot be achieved.

This paper deals with the problem of blending numerical and geometrical criteria in the decimation process of numerical simulation data sets. We propose three different approaches:

- Compression with priority to the simulation data,
- Compression based on a blending of geometrical and numerical criteria,
- Compression with priority to the geometry.

These approaches are illustrated on real world examples of mechanical phenomena simulations.

1 Introduction

This paper deals with the problem of blending numerical and geometrical criteria in the decimation process of numerical simulation data sets. Our method relies on two decimation algorithms that can be implemented independently of each other. The first is dedicated to the compression of the geometry and is based on a unique scalar parameter related to the chordal distance between the initial and the simplified model. The second decimation algorithm is dedicated to the numerical data attached to the geometry. It performs a wavelet-like decomposition of the numerical data that allows both a good approximation of the data on the simplified meshes, and the ability to recover original values when inverting the decimation process.

The main contribution of this paper is to present different ways of blending together the compression of the geometry with the compression of the data attached to the geometry. We propose three approaches that we think can contribute to the integration of Finite Element (FE) techniques into the design process:

- Compression with priority to the simulation data,
- Compression based on a blending of geometrical and numerical criteria,
- Compression with priority to the geometry.

These approaches are illustrated on real world examples of mechanical phenomena simulations.

Decimation approaches have been largely developed over the past years [1, 2, 3, 4, 5, 6, 7, 8] to set up mainly two categories of operators, i.e. vertex removal or edge collapse. In addition, the proposed approaches led to different concepts of control over the decimation process using energy criteria [1] or bounded deviation [3, 4, 5, 6].

These criteria and operators have produced different categories of decimators capable of restoring the initial shape of an object that has been arbitrarily triangulated. Bounding the deviation between the initial and decimated triangulations has appeared critical for many applications in the context of mechanical engineering and design. To this end, the approach by Véron and Léon [6, 7] uses a concept of error zone attached to each vertex of the initial triangulation to model a discrete envelope of variable distance around the object.

All these categories of approaches are based on the same similar concept of preserving, as much as possible, the shape of the initial object.

Wavelet encoding of numerical data has been extensively applied over the past years. Classical wavelet encoding schemes apply only on 1D data or 2D and 3D tensor-product data. Some generalizations have been proposed in the context of Computer Graphics, in order to define wavelet encoding scheme for arbitrary topological surface meshes [10,11]. However these approaches apply only on a restricted set of surface meshes, and some preprocess transform of the input mesh must be done before using these wavelet encoding schemes [12]. In the context of mechanical engineering and design, such a preprocess transform is clearly undesirable. The approach by Bonneau and Gerussi [13,14] is based on a generalization of the classical multiresolution analysis framework, and allows to handle numerical data attached to arbitrary planar or spherical triangular meshes. This approach has been extended to allow the case of numerical data attached to faces or vertices of a mesh that is decimated by a vertex removal process [15].

In the context of scientific visualization of simulation datasets, it is important to provide tools that fit into the design flow. This remark applies particularly to the field of simulation of mechanical phenomena using a Finite Element (FE) technique and contributes to the integration of FE techniques into the design process. Up to now, two distinct needs have been identified in this context:

- The engineer needs specific approaches to obtain models adapted to visualization capabilities of workstations and enabling him (resp. her) to get the best possible understanding of the

phenomenon simulated. Currently, the FE models become larger and larger and their visualization strictly relies on the straightforward visualization of these models. Even with high-end workstations, these models become difficult to visualize and don't allow the engineer to analyze and understand deeply the physical phenomenon because there no possibility of animation when transient of modal analyses are performed. As a result, a model dedicated to the visualization phase has appeared critical to produce a compressed model that still contains the significant information of the simulation while it is significantly compressed. Such a model becomes also a basis for co-located collaborative work during digital mock-up reviews to provide explanations to non specialist engineers,
- The engineer needs to collaborate with other engineers of the same skills during the design process through distant synchronous or asynchronous work. At present, such type of activity is not possible when FE models are large because they can't be sent through the mail and they are compatible with synchronous collaborative tools. Here, again the compression of such FE models becomes critical to exchange data over an intranet/internet network.

As a result, the objective is to present some approaches that lead to compression of simulation datasets through the use of control parameters that are compatible with the analysis activity performed by an engineer. The compression process should be transparent for the user and hence, automatic to produce a visualization model that can be effectively used for the analysis task.

2 Geometric compression through a decimation process

The purpose of this section is to review the main features of the geometric decimation process set up to perform a compression of a FE model.

2.1 Vertex and edge categories

First of all, a FE mesh defines a polyhedron that is often non-manifold; hence, the decimation process set up must be able to cope with such models. To this end, the nodes and vertices of the input polyhedron are classified in accordance with the following rules. The edges are classified according to:
- An edge which does not take part in the description of any face is classified as an isolated edge,
- An edge that takes part in the description of one face only is classified as a boundary edge. This edge holds for the boundary of a domain topologically equivalent to half a disc,
- An edge that takes part in the description of two faces only is classified as a surface edge. This edge is located in a domain topologically equivalent to a disc,
- An edge that takes part in the description of more than two faces is classified as a contact edge. Such an edge is the common boundary of more than two domains topologically equivalent to half discs.

A vertex is classified according to:
- A vertex connected to surface edges only is classified as a surface vertex if all the faces meeting at that vertex define one surface only, i.e. each vertex is associated with one and only one contour polygon) (see Figure 1a). Otherwise, this vertex is classified as an isolated vertex (see Figure 1d),
- A vertex connected to two boundary edges and to surface edges is classified as a boundary vertex if all the faces meeting at that vertex define one surface only (see Figure 1b), otherwise it is an isolated vertex (see Figure 1e),
- A vertex connected to at least one contact edge is classified as a contact vertex (see Figure 1c) except when it exists a face meeting at that vertex which owns two boundary edges meeting at that node. In this last case the vertex is classified as isolated (see Figure 1f),

- Finally, when a vertex is connected to more than two boundary edges or at least to one isolated edge, it is classified as an isolated vertex (see Figure 1g).

In order to carry on a brief description of the decimation principle that is used, only the key concepts of this principle are described.

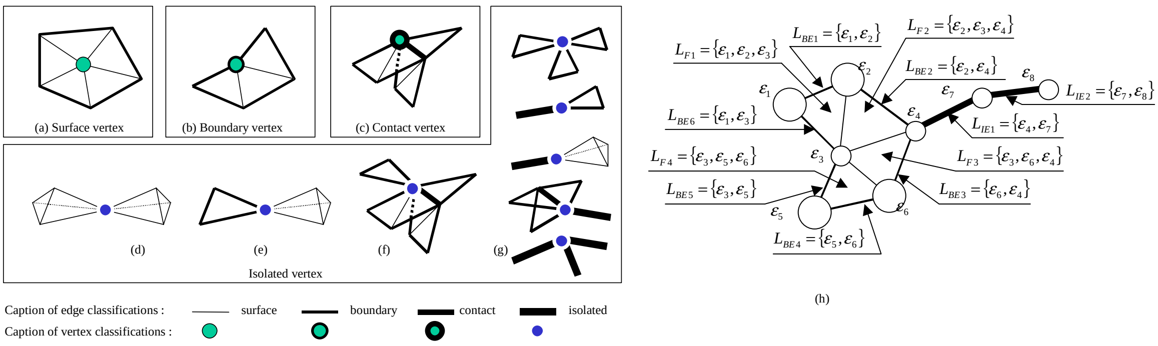

Figure 1: Illustration of vertex classifications (a to g). Error zones ε_i associated with the initial vertices and dependency lists assigned to faces L_{Fi}, boundary edges L_{BEj} and isolated edges L_{IEk} (h).

2.2 Main features of the decimation process

The approach proposed is based on an iterative vertex removal algorithm. First of all, the edges and vertices of the initial polyhedral model are classified in accordance with their local topological configuration as described previously. This classification is required to apply the appropriate selection criterion and vertex removal operators to each class of vertices. Then, the simplification treatment is initialized. A spherical error zone is assigned to each vertex of the initial model. The radius of these spheres is set up using values specified by the user and attached to different areas of the object. The set of error zones can be understood as a discrete envelope set up around the initial polyhedron where the decimated polyhedron must lie.

In the context of the present application, the radius of the spheres is constant and is the only parameter required to initiate the decimation process.

At each face, an inheritance process of the error zones is initialized to monitor the geometry restoration during the simplification process. The restoration criterion used is based on the measure of a geometric deviation between the initial and simplified models, i.e. the local chordal distance between the new faces and the error zone attached to the vertex removed. Afterwards, the simplification process starts and a loop is executed until no more candidate vertices can be removed. Different criteria based on discrete curvature approximations [9] are used to select the candidate vertex having the best probability of removal. Then an operator adapted to the classification of the candidate vertex is applied to create a new geometry from the contour polygon of this vertex, i.e. its star-polygon. To this end, different meshing techniques of 3D contour polygons, which take into account an approximation of the principal directions of curvature, are used in accordance with the local geometric configuration around the vertex to be removed. The geometry restoration criterion is then applied to determine whether the vertex can be removed or not. If the geometry of the initial model is correctly restored, the current model is updated using the previously created mesh of the 3D contour polygons and the possible topologic changes involved by this vertex removal are identified and managed.

2.3 Geometry restoration criterion

The geometric restoration process is based on the error zones assigned to the vertices and on an inheritance mechanism of these error zones attached to faces. At first, spherical error zones centered on each vertex of the input polyhedral model are generated. The radius of each sphere locally defines the maximum deviation accepted between the initial and simplified models and is the *only* parameter required for this process. This model can be either a two-manifold model or a non-manifold one.
Then, an inheritance process is used to monitor the geometric restoration of the object shape during the simplification process. This process is initialized using the input data. A dependency list of error zones, containing all the error zones participating to the local restitution of the object geometry, is assigned to each face of the model. For each face, its dependency list is initialized with the error zones attached to the vertices describing that entity. To complete the geometric restoration control, the same concept of dependency lists is also applied to the boundary and isolated edges of the model. Figure 1h illustrates the dependency list initializations on a simple initial model containing eight error zones ε_i .At the initialization stage, the error zones assigned to the dependency lists of each entity (face or edge) are solely composed of the zones located at their boundary vertices.
During each vertex removal procedure, the geometric restoration criterion is checked and the potential topological changes are managed using the following criteria and rules. The first step of the geometric restoration test merges the lists of the error zones of the faces and edges connected to the candidate vertex. This list L is created from the error dependency lists associated to the faces L_{Fi}, the boundary edges L_{BEj} and the isolated edges L_{IEk} meeting at the candidate vertex. The second step effectively tests the newly created re-meshing scheme to ensure that it intersects the error zones of all vertices, even the removed ones that are kept by the inheritance process during the simplification. The criterion used checks that each error zone ε_l of the list L previously created intersects at least either one face F_i or one boundary edge E_{Bj} or one isolated edge E_{Ik} of the newly created geometry, i.e. $\forall \varepsilon_l \in L$, $\exists\ F_i$ such that $F_i \cap E_l \neq \{\phi\}$ or $\exists\ E_{Bj}$ such that $E_{Bj} \cap E_l \neq \{\phi\}$ or $\exists\ E_{Ik}$ such that $E_{Ik} \cap E_l \neq \{\phi\}$. Hence, this criterion is based on sphere-triangle or sphere-edge intersection tests.
If the shape restoration test is successful, the model is updated. To this end, the newly created re-meshing scheme is locally inserted into the current model. Moreover, the dependency list of error zones assigned to each newly created (by the meshing process) face, boundary edge and isolated edge must be updated too. This updating process is carried out from the list L of error zones previously defined. Each error zone ε_i of this list is added to the dependency list L_{Fj} of the newly created face if it intersects this face F_j. In the same way, the error zone ε_i is added to the dependency list L_{BEk} (respectively L_{IEl}) of the newly created boundary edge BE_k (resp. isolated edge IE_l) if it intersects this edge.

3 Wavelet compression

This section very briefly describes the wavelet-like scheme that deals with the numerical data attached to the meshes. This scheme is based on previous works [13,14] about the simplification of piecewise constant or linear data on irregular planar or spherical meshes. The basic idea is to use vertex removal to simplify the mesh, and to best approximate the numerical data on the simplified mesh. The numerical error arising during this mapping process is stored in detail coefficients, that have been shown to generalize wavelet coefficients in some sense [13]. Computing the numerical data on the simplified mesh, and the detail coefficients, involves the evaluation of scalar products between piecewise constant or linear functions on the initial and the simplified mesh. The results of [13,14] have later been generalized in [15] in order to handle arbitrary surface meshes. To this end, a local planar projection is performed during each vertex removal (see figure 2a), and the results of [13,14] can be applied locally on the projected planar mesh.

In the context of the present paper, an important feature of this wavelet-like decomposition is that each removed vertex is associated to a scalar coefficient that measures the numerical error arising from its removal. This error coefficient associated to each removed vertex can then be used to blend geometric and numerical criteria during the decimation process.

4 Compression with priority to the simulation data

Because the purpose of the visualization model can be considered as the basis of the analysis process of a physical phenomenon, the engineer is interested in preserving roughly the shape of the initial mesh and then, concentrates on the values of the numerical simulation process and the areas where critical values are located.
In the context of mechanical engineering, this principle is set up using conceptually different areas. These areas are identified automatically from the numerical simulation values assigned at faces of the FE mesh. At present, the current approach uses constant values at each face of the FE mesh.
Prior to any visualization, the engineer can access the histogram of simulation values using the file output by the computation phase. According to Figure 2b, the values obtained are V, $V \in [V_{\min}, V_{\max}]$. Then, the engineer can specify a first threshold value $S_{\max}$, $S_{\max} < V_{\max}$ such that in the interval $]S_{\max}, V_{\max}]$ the simulation values should not altered by the decimation process to enable a rigorous analysis of the results. This interval defines the “area of interest” for the engineer and no compression should take place in this area. Therefore, this area is not modified from the geometric point of view as well as from the simulation one.
In addition, the engineer can specify a second threshold value $S_{\min}$, $S_{\min} < S_{\max}$, to define new areas where the decimation process will take place as a multi-criteria process. At present, the approximation criterion for the simulation data located around a candidate vertex is expressed as (see Figure 2c):

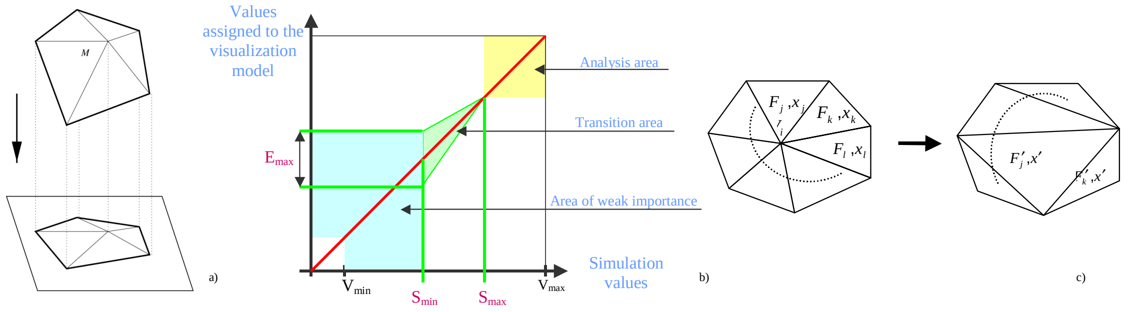

Figure 2: a) Local projection during vertex removal. b) Threshold values of simulation data used to control the compression process. c)Approximation of simulation data at a candidate vertex.

$$\sum_i^n x_i A_i = \sum_j^{n'} x' A'_j \qquad (1)$$

where x_j and A_j are the simulation value and the area of the face F_j and A'_j is the area of the face F'_j obtained after the vertex removal operation. x' is the constant value assigned to each face F'_j obtained when the vertex has been removed and x' can be obtained straightforwardly from eq. 1.
Within the transition area depicted on Figure 4, the simulation values attached to the visualization model must be kept into the interval $[V - E(V), V + E(V)]$, $V \in [S_{\min}, S_{\max}]$ where the function $E(V)$ is linear within the transition area. Hence, the transition area is characterized by the deviation over the simulation that the engineer can accept in the interval $[S_{\min}, S_{\max}]$. Such a behaviour is effectively described by two parameters $E_{\min}$ and $E_{\max}$ (see Figure 2b with a configuration where $E_{\min} = 0$).

As a result, the multi-criteria decimation approach is applied as follows:

- Vertices are selected using a geometric criterion based on discrete curvatures to ensure the preservation of the object shape,
- The contour of the vertex removed is replaced by a new set of faces with new simulation data attached to these faces,
- The geometry restoration criterion is checked (see the inheritance process described at section 2.2). If the new triangulation lies within the error zones of radius ε_g the remeshing scheme is validated otherwise this vertex is not removed,
- The approximation criterion for the simulation data is checked. If this approximation lies in the interval $[V_{init} - E(V_{init}), V_{init} + E(V_{init})]$, this criterion is satisfied and the candidate vertex is effectively removed otherwise it is not.

Finally, a third area is considered and designates a region of weak interest for the engineer (areas of low level of stress, …). In this area $V \in [V_{\min}, S_{\min}[$, the decimation process is applied with the geometric deviation criterion ε_g only and the simulation data are propagated without any further constraint in accordance to the process expressed by eq. 1.

As a result, the control parameters set up by the engineer are: ε_g, $S_{\min}$, $S_{\max}$, $E_{\max}$ and eventually $E_{\min}$ if a non-zero value is wanted. All these parameters are strictly based on the results of the simulation process and easily set up from the histogram of the results. Hence the compression is adaptative over the triangulation and driven by the simulation results. The ε_g parameter is also easily set by the engineer since it characterizes the chordal deviation between the initial mesh and the final triangulation.

To illustrate the effect of the simulation driven approach to set the control parameters, an example is provided at Figure 3 concerning a car body subjected to a modal analysis. This model is complex and non-manifold. It does not incorporate isolated edges and vertices but contact ones only. During the decimation process no topological change has been allowed. The simulation data visualized are the nodal displacements of the FE mesh. The mode displayed at Figure 3 is local to the beam placed under the roof. Figure 4 clearly shows the effect of the simulation data driven compression where red and yellow areas are critical for the analysis and dark blue ones are not relevant for the engineer.

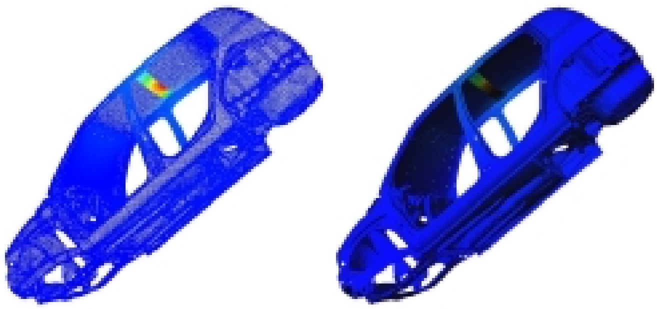

Figure 3: FE model of car body produced by a modal analysis (238 945 faces) (Courtesy Renault).

To illustrate more clearly the effect of the transition area, another vibration mode of the same structure is provided at Figure 5a. Here, significant displacement amplitudes are spread all over the structure and the mode is more global. Again, the red and yellow areas are kept unchanged and the green to cyan area reflect the transition area specified by the engineer. Figures 5b and 6 illustrate how the triangulation progressively increases in size as the colors move from green to cyan. The dark blue area is still the area where the geometric criterion only is active to validate the decimation process.

The above process is entirely automatic after the engineer has specified the thresholds and value and produces a significant compression of the model while preserving the meaningful information for the analysis phase carried out by the engineer.

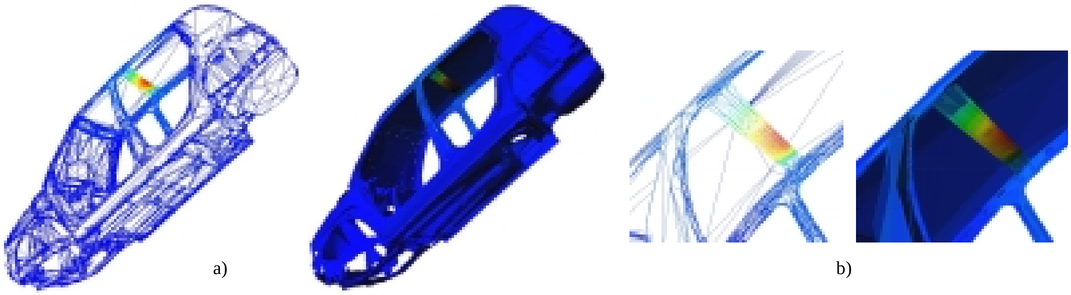

Figure 4: a) Visualization model of the vibration mode of the car body at Figure 3 (43 791 faces). ε_g =1cm. b) Detailed area of the visualization model.

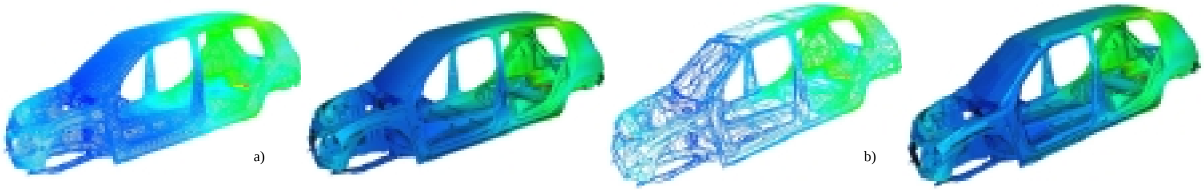

Figure 5: FE model of car body structure produced by a modal analysis (238 945 faces). The vibration mode is different from that of Figure 4. Visualization model of the mode of the car body (62 282 faces). ε_g =1cm (Courtesy Renault).

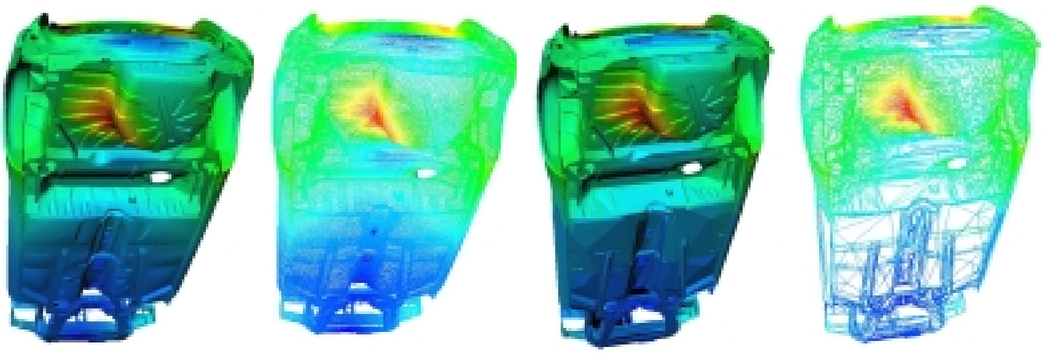

Figure 6: FE model (top) (238 945 faces) and visualization model (bottom) (62 282 faces) of a car body structure produced by a modal analysis (Courtesy Renault).

5 Compression through a weighted approach

In this section we describe the weighted approach for blending the compression of the geometry with the compression of the numerical data attached to the geometry.

In a numerical simulation data set, there is no relation between the complexity of the geometry and the regularity of the numerical data. If the geometry and the numerical values do not vary much in an area, then things are simple: this area can be simplified without perturbing much the initial data. If the geometry and the numerical data are both complex, then it is obvious that this area should not be modified at all. But if the geometry is complex and the numerical values are almost constant, or if the geometry is almost planar and the numerical values have a complex behavior, then one has to decide which way to go: either simplify the area, or do not modify it. Suppose for example that some part of

the mesh is exactly planar. Then a decimation based solely on the geometric criterion would remove all interior vertices in this area, thus resulting in a very poor quality of the numerical values inside this area.
In this section we propose to blend the geometric and the numerical criteria through a weighted approach. Both criteria are based on a scalar attached to each vertex. For the geometric decimation, this scalar is related to the error zone at that vertex. For the numerical simplification, this scalar is related to the error arising from the best-approximation mapping performed during the vertex removal. We can use an affine combination of these geometric and numerical criteria in order to decide in which order the vertices should be removed. Depending on the weights in this affine combination, the geometry or the numerical behavior of the data set will be preferred.
Figure 7 shows an example of two different decimations for two choices of weights. Notice in (b) how the numerical data is very poorly reproduced in planar areas of the object. After normalization of the error coefficients, we choose in (c) the weights 2/3, 1/3 respectively for the geometric and the numerical criteria. Notice how the numerical values are much better approximated, while the geometry is poorly reproduced, in particular around the cylindrical holes, or along the sharp edges.

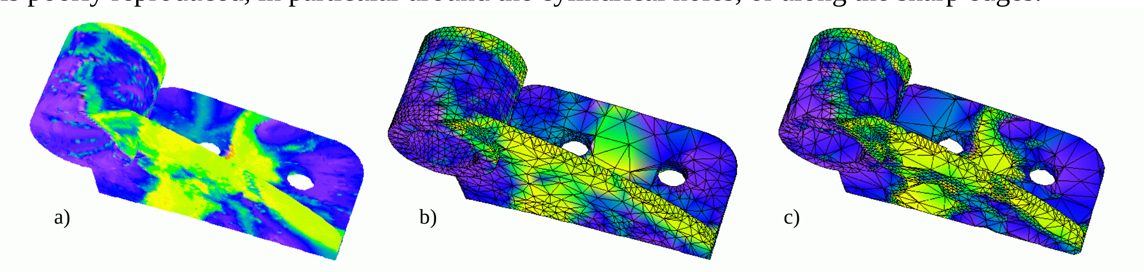

Figure 7: Bracket data set. (a) initial mesh, 50000 triangles; (b) decimation based on the geometric criteria, 3000 vertices; (c) decimation with weights 2/3 for the geometric criteria and 1/3 for the numerical criteria, 3000 vertices.

6 Compression based on a priority to the geometric criterion

As described at section 4, the simulation data driven compression of the FE model involves a concept of transition area where a multi-criteria decimation approach is applied. This decimation process sets the priority to the geometric criterion in the sense that the remeshing scheme is solely based on geometric criteria (discrete curvature criteria) and the validation of the remeshed area is subjected to the geometry restoration criterion first and to the restoration criterion for the simulation data afterwards. As depicted through the examples of Figures 3-7 of sections 4 and 5, the model compression based on priority to geometry or to a weighted relationship between simulation data and geometry leads to significantly different results. Preserving the object aspect can be obtained through a unique control parameter, , and produces a high compression rate of the FE model. The principle of the decimation approach allows the algorithm to use a variable distance between the initial FE model and the decimated model using error zones of variable radius. The radius variation could be indexed on the simulation values to provide a higher compression rate in areas of weak interest for the engineer. Such configurations have been already investigated for FE models preparation [9].

7 Conclusion

A set of approaches has been described and analyzed to perform the compression of FE simulation results. Simulation data driven compression has demonstrated its interest and applicability in industrial

configurations. Its adaptative behavior is an important factor to obtain automatically a meaningful visualization model from the histogram of the simulation process.
The weighted approach is based on an affine combination of a geometric error and a numerical error. Depending on the weights, the decimation will reproduce in priority either the geometry or the numerical data attached to the geometry.
The compression based on a priority to the geometric criterion highlights its importance in a context where the preservation of the initial geometry of the object seems critical. Effectively, it is not clear at present into which extent an engineer can feel comfortable to perform an analysis on a visualization model where the shape has significantly changed in areas where the simulation data is usually considered as critical by the engineer. Such a configuration needs to be investigated further.
Future work will focus on incorporating effectively the wavelet compression in the approaches described in sections 4 and 6, in order to improve the FE model compression by providing a more efficient transfer of simulation data during the decimation process. The use of textures is also direction of investigation to improve the compression of the model, apply the wavelet approach over a large part of the model and decouple the geometric decimation process from the simulation data transfer.

8 References

[1] H. Hoppe, T. DeRose, J. MacDonald, W. Stuetzle. Mesh Optimization, Computer Graphics, SIGGRAPH'93, pp 71-78, 1993.

[2] H. Hoppe. , J. C., Progressive Meshes, Computer Graphics, SIGGRAPH'96, pp 99-108, 1996.

[3] R. Klein, G. Liebich and W. Straber. Mesh reduction with error control, IEEE Visualization'96 Proceedings, pp 311-318, 1996.

[4] A. Guéziec. Surface simplification inside a tolerance volume, Technical report RC 20440, IBM Research Division, T J Watson Research Center, New York, USA, Mat, 1996.

[5] J. Cohen, A. Varshney, D. Manocha, G. Turk, H. Weber, P. Agarwal, F. P. Brooks, Jr. and W. V. Wright. Simplification Envelopes, in Computer Graphics Proceedings, ACM SIGGRAPH, August 4 - 9, pp 119 – 128, 1996.

[6] P. Véron, J-C Léon, Static polyhedron simplification using error measurements, CAD, 29(4): 287-298, 1997.

[7] P. Véron, J-C. Léon. Shape Preserving Polyhedral Simplification with Bounded Error, Computers & Graphics, 22(5): 565-585, 1998.

[8] E. Puppo, R. Scopigno. Simplification, LOD and Multiresolution – Principles and Applications, Eurographics'97 Tutorial Notes PS97 TN4, Eurographics association, 1997

[9] L. Fine, L. Rémondini, J-C. Léon. Automated Generation of FEA models through idealization operators, Int. Journal For Num. Methods In Eng., 49(1-2): 83-108, 2000.

[10] M. Lounsbery, T.D. DeRose, J. Warren, "Multiresolution Analysis for Surfaces of Arbitrary Topological Type", ACM Transactions on Graphics, vol. 16, no. 1, pp. 34-73, Jan. 1997.

[11] P. Schröder, W. Sweldens, „Spherical Wavelets: Efficiently Representing Functions on the Sphere", SIGGRAPH'95, pp. 161-172, Aug. 1995.

[12] M. Eck, T. DeRose, T. Duchamp, H. Hoppe, M. Lounsbery , W. Stuetzle, "Multiresolution Analysis of Arbitrary Meshes", SIGGRAPH'95, pp. 173-182, Aug. 1995.

[13] G.P. Bonneau, "Multiresolution Analysis on Irregular Surface Meshes", IEEE Transactions on Visualization and Computer Graphics, vol. 4, no. 4, pp. 365-378, 1998.

[14] G.P. Bonneau, A. Gerussi, « Level of Detail Visualization of Scalar Data Sets on Irregular Surface Meshes", IEEE Visualization'98, pp. 73-77, Oct. 1998.

[15] G.P. Bonneau, A. Gerussi, "Analysis of Scalar Datasets on Multi-Resolution Geometric Models", in Curve and Surface Fitting: Saint-Malo 1999, A. Cohen, C. Rabut, and L.L. Schumaker (eds.), Vanderbilt University Press., Nashville, pp. 209-218 (2000).